GAYATRI SAHU

M. Sc. IIIrd Semester

PAPER - IV

OPERATION RESEARCH

Seminar Topic :- GOAL PROGRAMMING

(i) Introduction

(ii) Formulation

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GOAL PROGRAMMING

Introduction :-

linear programming basically it the technique applicable only when there is a single goal (objective function), such as maximizing the profit or maximizing the cost or loss.

In such situation, we need a different technique that seek a compromise solution based on the relative importance of each objective. This technique is known as Goal programming.

OR

Defination :-

In earliar days, Profit maximization was considered as prime goal of management, But now a days decionion criteria for today's management is multidimensoional and decision process invole multiple gole.

CATEGORISATON OF GOAL PROGRAMMING

The goal programming problem may be categorised in terms of how the goals compare- non-pre-emptive or pre-emptive.

Non-pre-emptive :-

When all the goals are of roughly comparable importance, goal programming is known as non-pre-emptive.

Example :-

Given in the previous section is an example of non-pre-emptive goal programming problem.

In case of a pre-emptive goal programming, the goals are assigned priority levels. The goals are ranked from the most important (goal 1) to the least important (goal n) and the objective function coefficient assigned for the (deviational) variables representing goal i is P_i.

It is assumed that $P_1 >>> P_2 >>> P_3 \ldots >>> P_n$, where the symbol $>>>$ means 'more important than'.

In fact, P's are not assigned any actual any values.
Rather they are a convenient way of indicating that one goal is more important than the other.

These coefficients indicate that weight of goal 1 is much larger than weight 2, which in turn is much larger than that of goal 3 and so on.

FORMULATION OF LINEAR GOAL PROGRAMMING PROBLEM

Following are the major steps in the formulation of linear programming problem:

Step I :- Identify the decision variables of the key decision.

Step II :- Formulate all the objective or goals of the problem. These are generally determined by or implicitly placed on the decision maker.

determined by

(i) the desire of the decision maker
(ii) limited resources,
(iii) any other restriction(s) either explicitly or implicitly placed on the desire of the decision maker.

Step 3: - Reduce the number of goals by eliminating a few negli- negligibly important or redundant goals.

Step 4: - Express each goal in the form of constraint equation by introducing a negative and positive deviation variable (denoted by d_i^- and d_i^+ respectively) i.e.

* $G_i : f_i(x_1, x_2, \dots x_n) - d_i^+ + d_i^- = b_i ;$

$i = 1, 2, \dots m$

where

d_i^- = negative deviation from ith goal (underachievement)

d_i^+ = positive deviation from ith goal (overachievement)

Step 5 : Assign the goals to priority levels. All absolute goals (i.e. either $d_i^- = 0$ or $d_i^+ = 0$) if any exist are used assigned to top priority.

Remark : -

(1) Since both of the under-achievement are over-achievement goal cannot be achieved simultaneously, one or both of the deviational variable d_i^- and d_i^+ must be zero in the solution. In other words, at optimality, if one assume a positive value in the solution, the other must be zero. and vice-versa.

(2) The deviational variable d_i^+ (called surplus variables in linear programming) is removed from the objective function of GP when overachievement is acceptable. Similarly, if underachievement is acceptable, d_i^- (called slack variables in LP) is removed from the objective function of G.P.

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Sol.) A firm product (produces) two product say, X and Y. Product X sells for a net profit of Rs. 80 per unit, while product Y sells for a net profit of Rs. 40 per unit. The goal of the firm is to earn Rs. 900 in the next week. Also, the managment want to achieve sales volume for the two products close to 17 and 15 respectively. Formulate this problem as a goal programming model.

Formulation :-

Let x_1 and x_2 denote the number of units of product X and Y respectively. The linear programming formulation of the problem is:

$$\text{maximize } z = 80x_1 + 40x_2$$

subject to the constraints

$$x_1 \leq 17, \; x_2 \leq 15 \text{ and } x_1, x_2 \geq 0$$

Since the goal of the pertains of profit attainment with a target established at Rs. 900 per week. the constraints of the problem can be stated as:

$80x_1 + 40x_2 = 900$

$x_1 \leq 17$ and $x_2 \leq 15$

The problem can now be formulated as goal programming model as follows

Minimize $Z = d_1^- + d_1^+ + d_2^- + d_3^-$

Sub. to constraints

$80x_1 + 40x_2 + d_1^- - d_1^+ = 900$

$x_1 + d_2^- = 17$

$x_2 + d_3^- = 15$

$x_1, x_2, d_1^-, d_1^+ \geq 0$.

where d_2^- and d_3^- represent underachievements of sales volume for product A and B respectively.

d_1^- = under-achievement of the profit goal of Rs. 900

d_1^+ = over-achievement of the profit goal of Rs. 900.

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Ex4 Consider the following L.P.P. :

Maximize $z = x_1 + x_2$ subject to the constraint

$3x_1 + 2x_2 \le 12$

$x_1 + x_2 \ge 8$

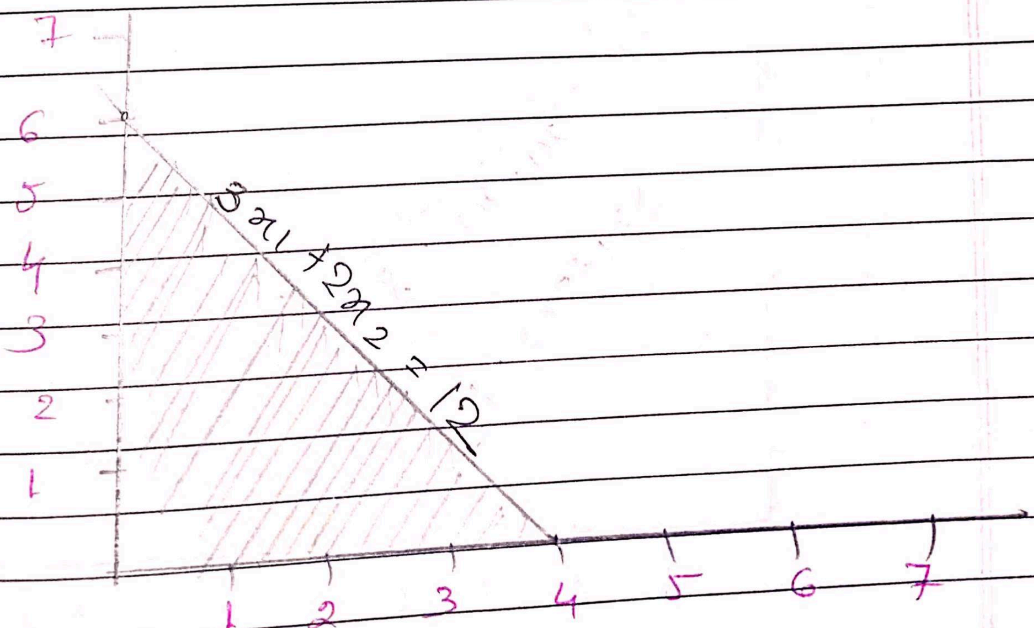
$-x_1 + x_2 \ge 4$

and

$5x_1 \le 10$

$x_1, x_2 \ge 0$

Solution :- Draws the inequalities them as a lines on the x_1, x_2 axis



$3x_1 + 2x_2 = 12$

Put $x_1 = 0 \Rightarrow 0 + 2x_2 = 12$

$\boxed{x_2 = 6}$

put $x_2 = 0$

$3x_1 + 0 = 12$

$\boxed{x_1 = 4}$

Now we draw the line (2)

$x_1 + x_2 = 8$

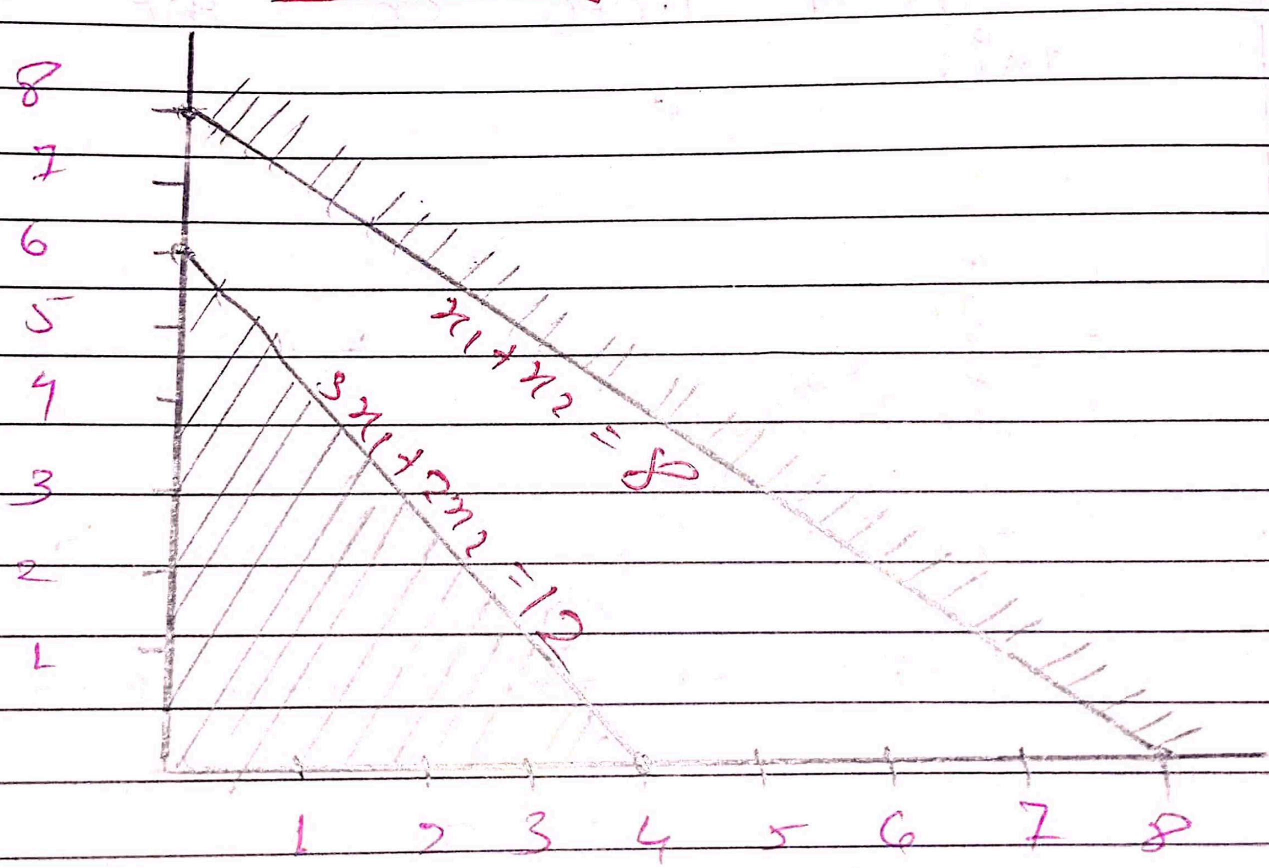
put $x_1 = 0$

$\boxed{x_2 = 8}$

put

$x_2 = 0$

$\boxed{x_1 = 8}$



shaded point is :

$x_1 + x_2 \geq 8$

Now we draw the line (3)

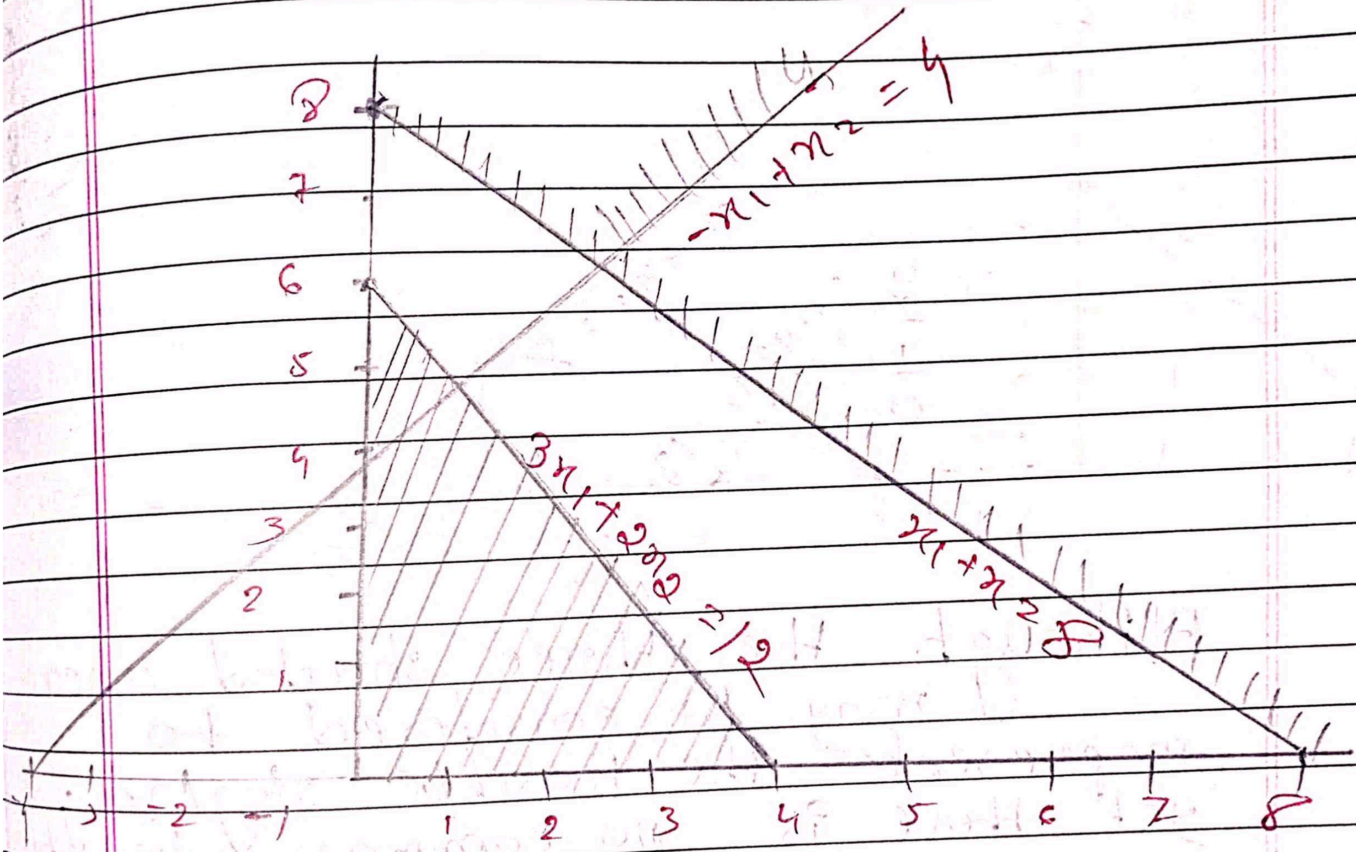
$-x_1 + x_2 = 4$

put $x_1 = 0$

$\boxed{x_2 = 4}$

put $x_2 = 0$

$\boxed{x_1 = -4}$



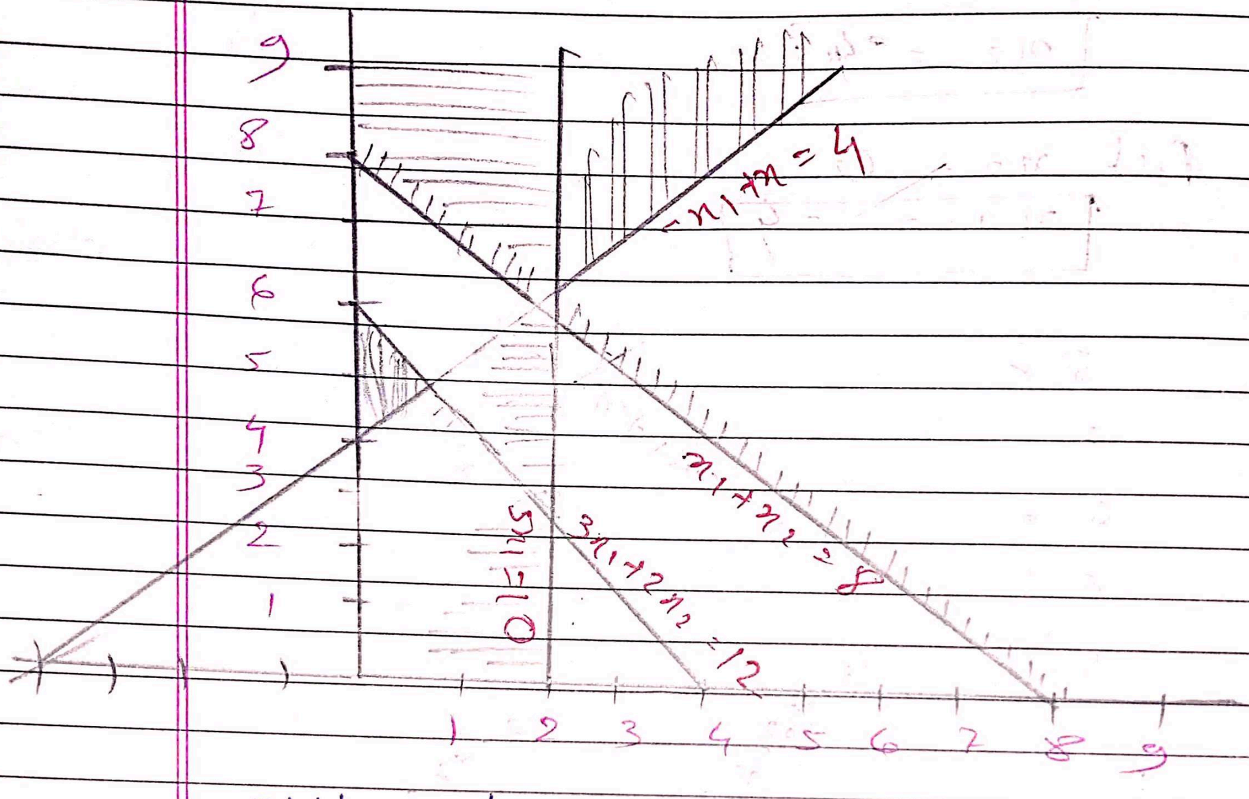
shaded point

$-x_1 + x_2 \geq 4$

Now the draw next line

$5x_1 = 10$

$\boxed{x = 2}$



Although the three shaded areas may be considered to represent the feasible region. yet there is no common feasible region. This is so because we cannot find any point (x_1, x_2) to lie on both these shade region.

This problem cannot be solved by the usual by the usual linear programming procedure.

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PAPER IV - OPERATION RESEARCH

SEMINAR TOPIC - GRAPHICAL GOAL ATTAINMENT METHOD

GRAPHICAL GOAL ATTAINMENT METHOD

The graphical goal attainment method is based on a well-defined set of logical steps. Following this systematic procedure, the given linear goal programming problem can be solved with a minimum amount of computational effort. The procedure may be summarised as follows:

Step 1. Formulate the appropriate linear goal programming problem.

Step 2: Construct the graph of all the objectives (goals) in terms of the decision variables as follows:

Treat each goal as if it were an equation with deviation (positive as well as negative) variables all set equal to zero, and for each goal equation arbitrarily select two sets of points. Plot each set of points and connect them with a straight line. Indicate the positive and negative deviations range by arrows → or ← for each goal line.

d_1^- = ~~under achievement~~

d_1^- - under utilization (idle time) of production capacity.

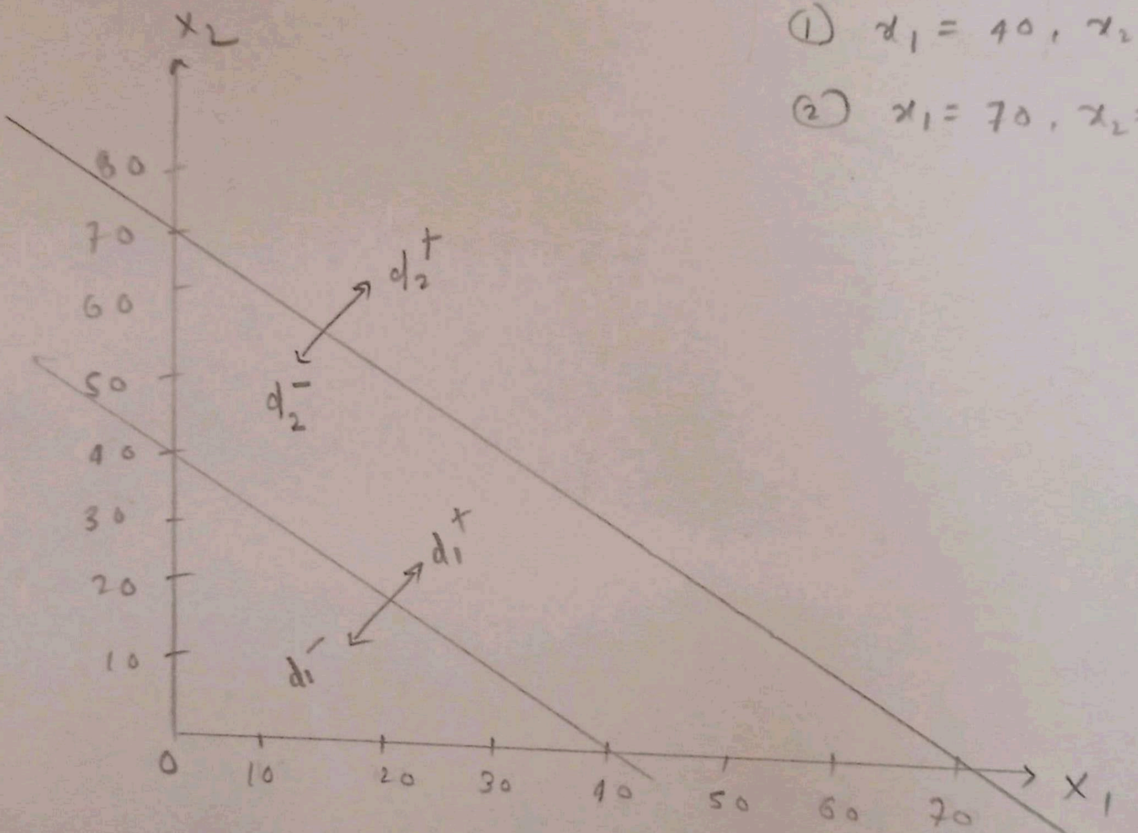
d_1^+ - over time operation of production capacity.

Step 3 :- Identify the goal line corresponding to the goal having top (highest) priority. Locate the feasible (solution) region with respect to this goal at the first priority level.

Step 4 :- Move to goal (s) having the next highest priority and determine the best solution (s) space with respect to the goal (s) at this next highest priority level provided that "best" ~~in~~ solution(s) do not degrade the solution (s) already achieved for higher priority goals.

Step 5. Repeat step 4 until all priority levels have been investigated.

Step 6 :- Identify the optimal solution which corresponds to the most acceptable "best" value, located in Step 5.



① $x_1 = 40, x_2 = 40$

② $x_1 = 70, x_2 = 70$

$$Z = G_1(d_3^+ + d_4^+) + G_2 d_1^+ + G_3 d_2^- + G_4\left(d_3^- + \frac{3}{2} d_4^-\right)$$

subject to the constraint

G_1 : $x_1 + x_2 + d_1^- + d_1^+ = 40$

G_2 : $x_1 + x_2 + d_2^- - d_2^+ = 100$

G_3 : $x_1 + d_3^- - d_3^+ = 30$

G_4 : $x_2 + d_4^- - d_4^+ = 15$

$x_i, d_i^-, d_i^+ \geq 0$.

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PAPER IV :- OPERATION RESEARCH

SEMINAR TOPIC :- GRAPHICAL GOAL ATTAIMENT METHOD (EXAMPLE)

808. Solve the following linear goal programming problem graphically:

Find x_1 and x_2 so as to:

Minimize $z = G_1(d_3^+ + d_4^+) + G_2 d_1^+ + G_3 d_2^- + G_4(d_3^- + \frac{3}{2} d_4^-)$

and satisfie goals:

$G_1 : x_1 + x_2 + d_1^- + d_1^+ = 40$

$G_2 : x_1 + x_2 + d_2^- + d_2^+ = 100$

$G_3 : x_1 + d_3^- - d_3^+ = 30$

$G_4 : x_2 + d_4^- - d_4^+ = 15$

$x_i, d_i^-, d_i^+ \geq 0$ for all $i = 1, 2, 3, 4$

The goals have been listed in order of priority.

Solⁿ:- ① $x_1 + x_2 = 40$

$x_1 = 0, x_2 = 40$

$x_2 = 0, x_1 = 40$

x_1	0	40
x_2	40	0

② $x_1 + x_2 = 100$

$x_1 = 0, x_2 = 100$

$x_2 = 0, x_1 = 100$

x_1	0	100
x_2	100	0

③ $x_1 = 30$

④ $x_2 = 15$

$$x_1 + x_2 = 40$$
$$x_2 = 15$$
$$x_1 + 15 = 40$$
$$x_1 = 25$$
$$[x_1 = 25, x_2 = 15]$$

The graph of various goal equations indicating clearly various deviations to be minimised is shown in fig. 8.4. we begin by attempting to satisfy highest priority goal (G_1). This will be attained by minimising $d_3^+ + d_4^+$ and can be achieved by setting $d_3^+ = d_4^+ = 0$. The feasible region satisfying the goal (G_1) at priority level one is shown shaded in Fig 8.5. Here, both d_3^+ and d_4^+ are set equal to zero.

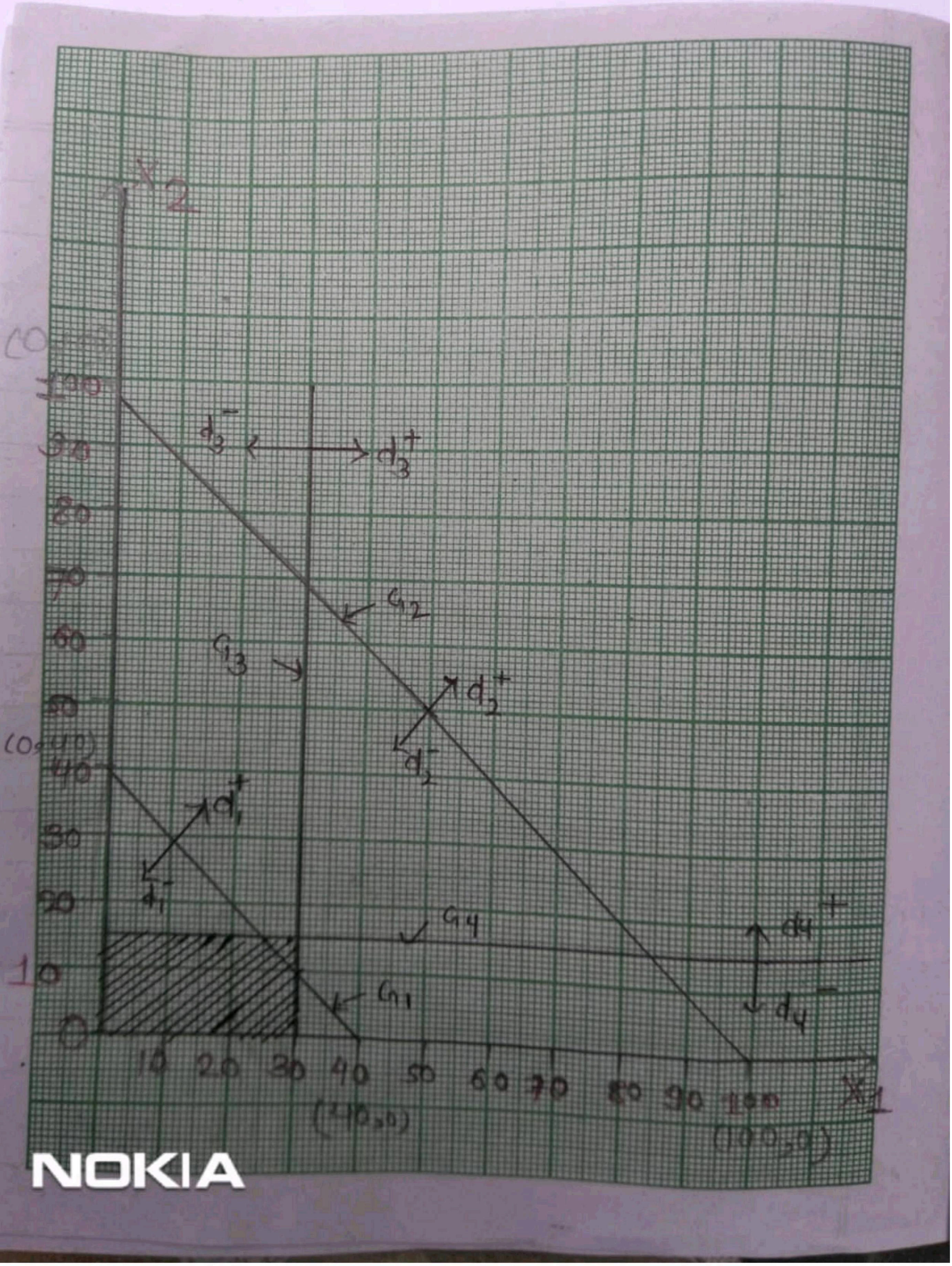
Now we attempt to satisfy priority level 2 without degrading the solution attained at priority level 1. since we are to minimise d_1^+, we may set $d_1^+ = 0$ without degrading the solution attained at the previous higher priority level (i.e. d_3^+ and d_4^+ are still zero). The feasible region at priority level 1 and 2 is shown shaded in Fig 8.5. we now attempt to achieve priority level 3. To attain it we must minimize

d_2. However d_2^- can not be set to zero as this would degrade our solution previously attained at both priority levels 1 and 2. Since G_1 and G_2 goal lines are parallel, the feasible region minimising d_2^- without degrading the previously attained higher priority goals is given by the line segment AB in Fig 8.5.

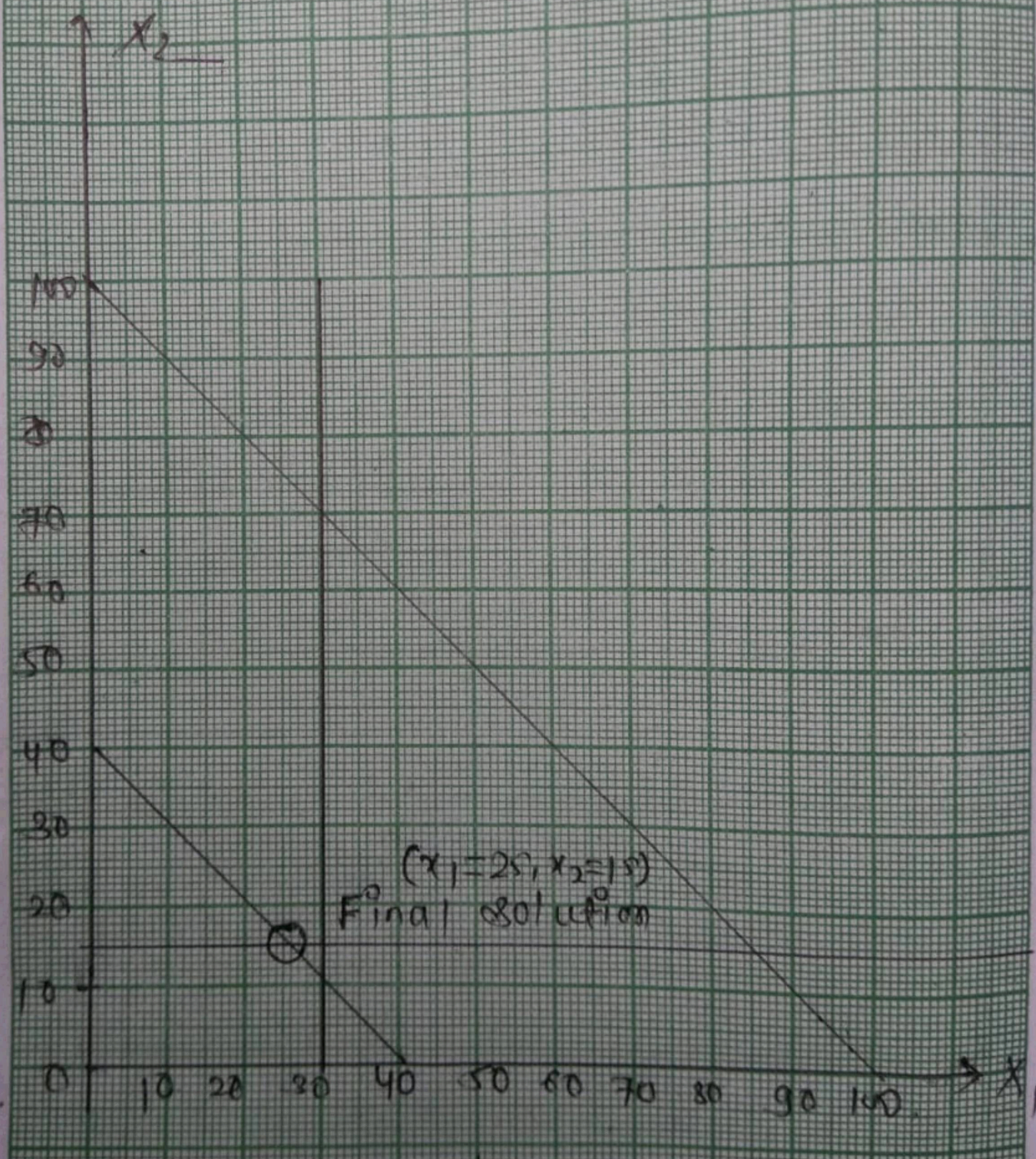
Finally to achieve G_4, at priority level 4, we must minimise $d_3^+ d_4^+$ we notice that d_4^+ is to be considered $\frac{3}{2}$ times more important than d_3^+. Thus the final solution without degrading the previously attained solution is attained at the point A when we set $d_4^- = 0$ (note that we do not set $d_3^- = 0$ because of the priority of d_4^- over d_3^-). Consequently, the final solution is the point shown in Fig 8.6. Thus

$$x_1 = 25, \; x_2 = 15, \; z^\circ = (0, 0, 60, 5)$$

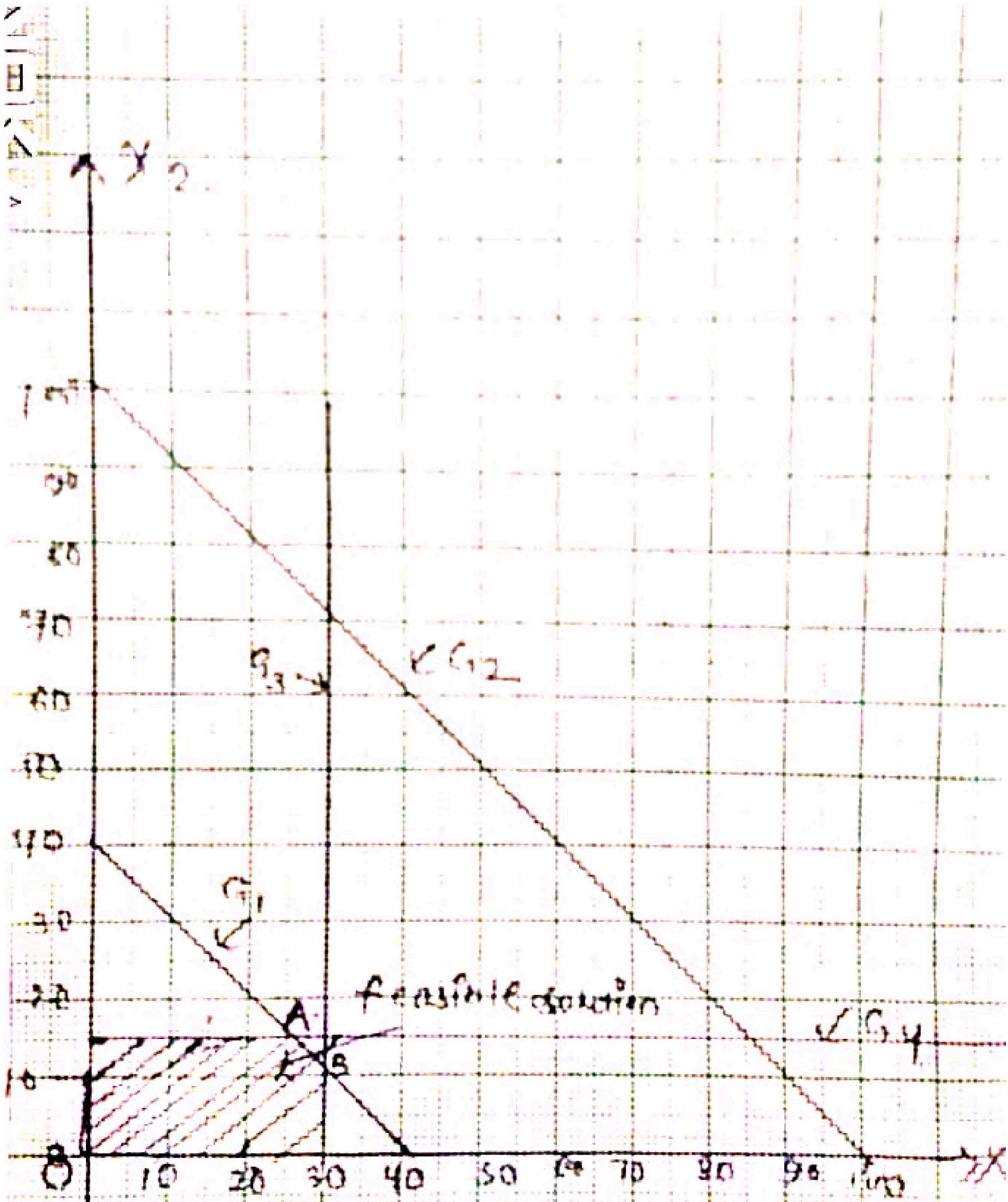
It is clear from the optimal values that the first and second priority goals have been attained completely while the third & four priority goals are attained as best as possible



X_2
100
90
80
70
60
50
(0,40)
40
30
20
10
0
10 20 30 40 50 60 70 80 90 100
X_1
(40,0)
(100,0)
d_3^-
d_3^+
G_2
G_3
d_2^+
d_2^-
d_1^+
d_1^-
G_4
d_4^+
d_4^-
G_1
NOKIA



 All priority levels



Priority levels 1, 2, 3

$$x_1 + x_2 + d_1^- + d_1^+ = 40$$
$$40 + d_1^- = 40$$
$$[d_1^- = 0]$$

$$x_1 + x_2 + d_2^- = 100$$
$$40 + d_2^- = 100$$
$$[d_2^- = 60]$$

$$x_1 + d_3^- - d_3^+ = 30$$
$$\cancel{x}\ 25 + d_3^- = 30$$
$$[d_3^- = 5]$$

Minimize $z = G_1(d_3^+ + d_4^+) + G_2 d_1^+ + G_3 d_2^- + G_4\left(d_3^- + \frac{3}{2} d_4^-\right)$

$$z^0 = (0, 0, 60, 5)$$

- Sneha Verma
- Msc 3rd Sem
- Operations Research

8.5 Simplex Method for Goal programming problem.

SIMPLEX METHOD FOR GOAL PROGRAMMING PROBLEM

By SNEHA VERMA

the major steps of the simplex method for the linear programming problem are :

Step-1 : Identify the decision variables of the key decision and formulate the given problem as linear goal programming problem.

Step-2 : Determine the initial basic feasible soln and setup initial simplex table. Compute z_j and $z_j - c_j$ values separately for each of the ranked goals : $P_1, P_2 \ldots$ and enter at the bottom of simplex table.

These are shown from bottom to top i.e first priority goal 'P_1' is shown at the bottom and least priority goal at the top.

Step-3 : Examine $z_j - c_j$ values in the P_1-row first.
If all $(z_j - c_j) \leq 0$ at the highest priority levels, then the optimum solution has been obtained.
If atleast one $(z_j - c_j) > 0$ at a certain priority level and there is no negative entry at a higher unachieved priority levels, in the same column, then the current soln is not optimum.

Step-4 : If the target values of each goal in the soln column (XB) is zero, the current soln is optimum.

Step-5 : Examine the positive values of $(z_j - c_j)$ of the highest priority (P_1) and choose the largest positive value. The column corresponding to this value become the key column. Otherwise move to the next higher priority (P_2) and select the largest positive value of $(z_j - c_j)$ for deteimining the key column.

Step-6 : Deteimine the key row and leading element in the same way as in simplex method.

Step-7 : Any positive value in the $(z_j - c_j)$ row which has negative $(z_j - c_j)$ under any lower priority rows are ignored. It is because deviations from higher priority goal would be increased with the entry of this variable in the basis.

Example 814

A textile company produces two types of materials 'A' and 'B'. The material A is produced by direct orders from furniture manufacturers. The material B is distributed to retail fabric stores. The avg production rate for materials A and B are identical 1000 m/hr. By running two shifts the operational capacity of the plant is 80 hr per week.

The marketing department reports that the maximum estimated sales for the following week is 70,000 meters of material A and 45000 m. of material B. Acc to the accounting department the profit from a meter of material A is Rs 5.00 and from a meter of material B is Rs 3.00.

The management of the company decided that a stable employment level is a primary goal for the firm. Therefore whenever there is demand exceeding normal production capacity, management simply expands production capacity by providing overtime. However management feels that the overtime of the plant of more than 10 hr per week should be avoided bcoz of the accelerating costs. The management has the following goals in order of their importance:

P_1 : Avoid any underutilization of production capacity

P_2 : Limit the overtime allowed for the plant operation to 10 h/week.

P_3 : Achieve the sales target of 70,000 m of material A and 45,000 m of material B.

P_4 : Minimize overtime operation of the plant as much as possible.

Soln

Formulation:

production hour constraint

The production capacity is limited to 80 hrs at present by running two shifts. The management may allow overtime if necessary. Thus the production hour const. may written as:

$$x_1 + x_2 + d_1^- - d_1^+ = 80$$

where x_1 = No. of hr used for producing material A.
x_2 = No. of hr. used for producing material B.
d_1^- = Amount of underutilization of available normal production hr.
d_1^+ = overtime beyond normal production hr. allowed by the management.

Sales Constraints

Maximum sales for material A and B are set at 70,000 and 45,000 respectively. Hence it is assumed that overachivement of sales beyond max limit is impossible then sales constraint is:

$$x_1 + d_2^- = 70{,}000 \Rightarrow x_1 + d_2^- = 70$$

$$x_2 + d_3^- = 45{,}000 \Rightarrow x_2 + d_3^- = 45$$

where d_2^- and d_3^- are underachievement of sales target of material A and B respectively.

Overtime Constraint.

The overtime constraint is

$$d_1^+ + d_4^- - d_4^- = 10$$

where d_4^- = amount of underutilize hours betn the actual overtime allowed and 10 hr of overtime.
d_4^+ = amount of overtime in excess of allowed 10 hr.

Both -ve and +ve deviations are included from allowed 10 hrs of overtime, bcoz the actual overtime can be less than, equal to or even more than 10 hrs.

Substituting d_1^+ in production hr constraint we get

$$x_1 + x_2 + d_4^- - d_4^+ = 90$$

Achievement function

Since, Sales goal of material is equally important, the profit ratio (5:3) ~~is equally distributed~~ betn these two will be considered as different weights the achievement funcn, therefore is:

$$\text{Minimize } z = P_1 d_1^- + P_2 d_4^+ + 5P_3 d_2^- + 3P_3 d_3^- + P_4 d_1^+.$$

The complete goal programming is:

Minimize $z = P_1 d_1^- + P_2 d_4^+ + 5P_3 d_2^- + 3P_3 d_3^- + P_4 d_1^+$

Subject to the constraints

$$x_1 + x_2 + d_1^- - d_1^+ = 80$$

$$x_1 + d_2^- = 70$$

$$x_2 + d_3^- = 45$$

$$x_1 + x_2 + d_4^- - d_4^+ = 90$$

and $x_1, x_2, d_1^+, d_1^-, d_2^-, d_3^-, d_4^-, d_4^+ \geq 0$

Solution by simplex table.

Initial Simplex table is:

		$c_j \rightarrow$	0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
C_B	Basic variable	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_4^+
P_1	d_1^-	80	1	1	1	0	0	0	-1	0
0	d_4^-	90	1	1	0	0	0	1	0	-1
$5P_3$	d_2^-	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	1	0	0	0.
$z_j - c_j$	P_4	0	0	0	0	0	0	0	-1	0
	P_3	485	5	3	0	0	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	-1
	P_1	80	$\boxed{1}$	$\boxed{1}$	0	0	0	0	-1	0

Using the simplex method for the calculation of z-values.

$$z = C_{B_1} x_{B_1} + C_{B_2} x_{B_2} + C_{B_3} x_{B_3} + C_{B_4} x_{B_4}$$

$$= P_1 \times 80 + 0 \times 90 + 5 \times P_3 \times 70 + 3P_3 \times 45$$

$$= \underline{80P_1 + 485 P_3}.$$

$\Rightarrow$ $P_1 = 80$, $P_2 = 0$, $P_3 = 485$ and $P_4 = 0$ in x_B colⁿ

this represent unachieved portion of each goal.

$z_j - c_j$ of each colᵐ is calculated as follows.

$$z_1 - c_1 = [P_1 \times 1 + 0 \times 1 + 5P_3 \times 1 + 3P_3 \times 1] - 0 \quad = P_1 + 5P_3.$$

$$z_7 - c_7 = P_1 \times (-1) + 0 \times 0 + 5P_3 \times 0 + 3P_3 \times 0 - P_4 \quad = -P_1 - P_4$$

First iteration:

In P_1 row, $z_1 - c_1$ and $z_2 - c_2$ are most positive $(=1)$ and same, so we check next priority levels in x_1 and x_2 columns. We observe that priority P_3 has the largest net evaluation '5' under x_1-colᵐ. Thus x_1 becomes the key column.

Now $\min\left\{\frac{x_{iB}}{y_{ij}}, y_{ij} > 0\right\} = \min\left\{\frac{80}{1}, \frac{90}{1}, \frac{70}{1}\right\} = 70$. Therefore

d_2^- leaves the basis and 1 becomes leading element

So, converting all other element zero of column x_1 we get

	$c_j \rightarrow$		0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
CB		x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_4^+
P_1	d_1^-	10	0	1	1	-1	0	0	-1	0
0	d_4^-	20	0	1	0	-1	0	1	0	-1
0	x_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	1	0	0	0
$z_j - c_j$	P_4	0	0	0	0	0	0	0	-1	0
	P_3	135	0	3	0	-5	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	-1
	P_1	10	0	$\boxed{1}$	0	-1	0	0	-1	0

$$Z = P_1 \times 10 + 0 + 20 + 0 + 70 + 3P_3 \times 45 = \underline{10P_1 + 135P_3}$$

Second iteration: $z_2 - c_2 > 0$ at P_1-level, so x_2 becomes key column. Proceeding the usual mannar.

$$\min\left\{\frac{x_{iB}}{y_{ij}}, y_{ij} > 0\right\} = \min\left\{\frac{10}{1}, \frac{20}{1}, \frac{45}{1}\right\} = 10$$

row one becomes the key row i.e d_1^- leaves the basis, 1 becomes leading element. So revised simplex Table is

	$c_j \rightarrow$		0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
CB		x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_4^+
0	x_2	10	0	1	1	-1	0	0	-1	0
0	d_4^-	10	0	0	-1	0	0	1	1	-1
0	x_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	35	0	0	1	1	1	0	1	0
	P_4	0	0	0	0	0	0	0	-1	0
	P_3	105	0	0	-3	-2	0	0	3	0
	P_2	0	0	0	0	0	0	0	0	-1
	P_1	0	0	0	-1	0	0	0	0	0

From the above table, $x = 70$, $x_2 = 10$, we observe that 70,000 m of A and 10,000 m is sufficient of achieve first second and fourth goal. Also $d_3^- = 35$ suggest that 35,000 m of material is not achieved.

Also, our first two priorities (P_1 and P_2) is achieved. and P_3 is not achieved. To improve current sol^n. We check the net evaluation at P_3 priority row.

In P_3. $(z_7 - c_7) > 0$ (3), so d_1^+ becomes the key column proceeding as usual, d_4^- leaves the basis and 1 becomes leading element [of 2nd row].

			0	0	P_1	$5P_3$	$3P_3$	0	P_4	P_2
CB		x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_4^+
0	x_2	20	0	1	0	-1	0	1	0	-1
P_4	d_1^+	10	0	0	-1	0	0	1	1	-1
0	x_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	25	0	0	0	1	1	-1	0	1
	P_4	10	0	0	-1	0	0	1	0	-1
	P_3	75	0	0	0	-2	0	-3	0	3
	P_2	0	0	0	0	0	0	0	0	-1
	P_1	0	0	0	-1	0	0	0	0	0

From the above Table we obseve that, In P_3-priority level there is a +ve value in d_4^+ column. But since there is a negative value in higher priority P_2 level (−1) in the same column, This can not become the key column.

Similarly in P4 row there is a positive value '1' corresponding to d_4^- column. But since higher priority P_3 has a −ve value in same column, this cannot become key column.

Hence the optimum solution is

$$x_1 = 70, \quad x_2 = 20, \quad d_1^+ = 10, \quad d_3^- = 25$$

$$d_1^- = d_2^- = d_4^- = d_4^+ = 0$$

Payal Sahu

MSC IIIrd Semester

Paper- IV

Operations Research

SEMINAR TOPIC :-

8:5 Simplex Method For Goal Programming Problem

PAYAL SAHU

SIMPLEX METHOD FOR GOAL PROGRAMMING

Some important things :

- "A key idea in goal programming is that one goal is more important than another. priorities are assigned to each deviationdl".

- priority 1, is infinitely more important than priority 2, which is infintely more importance that the next goal, and soon.

- Deviational variables represent overachieving or underachieving the desired level of each goal.

  - d^+ Represents <u>overachieving</u> level of the goal

  - d^- Represents <u>underachieving</u> level of the goal.

Que :- A Manufacturing firm produces two types of products A and B. requires an average of one hour in the plant. The plant has a normal production Capacity of 400 hours a month. The marketing department of the firm reports that because of limited market, the maximum number of products A and B that can be sold in a month are 240 and 300 respectively. The net profit for the Sale of product A and B are Rs. 800 and Rs. 400 respectively the manager of the firm has set the following goals arranged in the order of importance.

P_1 : He wants to avid any Under-utilization of normal production Capacity

P_2 : He wants to sall maximum possible units of product A and B. since the net profit from the Sale of product A is twice the amount of product B Therefore, the manager has twice as much desire to achive sales for product A as for product B.

P_3 : He want to minimize the over-time operation of the plant as mush as possible.

formulate and solve the given problem by Using simplex Method

Solution:

formulation of GP

- Let x_1 & x_2 be the number of Units of product A & B to be produced

- Since overtime operation is allowed, the plant capacity Constraint can be expressed as:

$$x_1 + x_2 + d_1^- - d_1^+ = 400 \quad \text{(Capacity constraint)}$$

- $d_1^- \Rightarrow$ Under utilization (idle time) of production capacity

- $d_1^+ \Rightarrow$ over time operation of production Capacity

- Since the sales goals are maximum possible sales volume; so sales constraints can be expressed as;

$$x_1 + d_2^- = 240 \quad \text{and} \quad x_2 + d_3^- = 300$$

(Sales Constraints)

- d_2^- and d_3^- are the under achievment of the sales goals for product A and B.

- Goal programming model becomes :-

Minimize $Z = P_1 d_1^- + P_2 (2d_2^- + d_3^-) + P_3 d_1^+$

$$Z = P_1 d_1^- + 2P_2 d_2^- + P_2 d_3^- + P_3 d_1^+$$

Subject to : $x_1 + x_2 + d_1^- - d_1^+ = 400$

$$x_1 + d_2^- = 240$$

$$x_2 + d_3^- = 300$$

$$x_1, x_2, d_1^-, d_2^-, d_3^-, d_1^+ \geq 0$$

Simplex method for GP problems :

Enter

C_B	Basic Variable	X_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+	Ratio
		$C_j \rightarrow$	0	0	P_1	$2P_2$	P_2	P_3	
P_1	d_1^-	400	1	1	1	0	0	−1	400/1=400
$2P_2$	Leave $\rightarrow d_2^-$	240	1	0	0	1	0	0	240/1=240
P_2	d_3^-	300	0	1	0	0	1	0	
$(Z_j - C_j)$	P_3	0	0	0	0	0	0	−1	
	P_2	780	2	1	0	0	0	0	
	P_1	400	1	1	0	0	0	−1	

Most positive

Using Simplex method for the Calcutions of Z Value We get

$$Z = C_{B1}X_{B1} + C_{B2}X_{B2} + C_{B3}X_{B3}$$

$$= P_1 \times 400 + 2P_2 \times 240 + P_2 \times 300$$

$$= 400P_1 + 480P_2 + 300P_2$$

$$= 400P_1 + 780P_2$$

The Values $\boxed{P_1 = 400}$, $\boxed{P_2 = 780}$, $\boxed{P_3 = 0}$ in the X_B column of the first iteration represent the

Unachieved portion of each goal

The $(z_j - c_j)$ value for each column is calculated as follows:

$z_1 - c_1 = P_1 \times 1 + 2P_2 \times 1 + P_2 \times 0 - 0$

$\boxed{z_1 - c_1 = P_1 + 2P_2}$

$z_2 - c_2 = P_1 \times 1 + 2P_2 \times 0 + P_2 \times 1 - 0$

$\boxed{z_2 - c_2 = P_1 + P_2}$

$z_3 - c_3 = P_1 \times 1 + 2P_2 \times 0 + P_2 \times 0 - P_1$

$= P_1 - P_1$

$\boxed{z_3 - c_3 = 0}$

$z_4 - c_4 = P_1 \times 0 + 2P_2 \times 1 + P_2 \times 0 - 2P_2$

$= 2P_2 - 2P_2$

$\boxed{z_4 - c_4 = 0}$

$z_5 - c_5 = P_1 \times 0 + 2P_2 \times 0 + P_2 \times 1 - P_2$

$= P_2 - P_2$

$\boxed{z_5 - c_5 = 0}$

$z_6 - c_6 = +P_1 \times -1 + 2P_2 \times 0 + P_2 \times 0 - P_3$

$\boxed{z_6 - c_6 = -P_1 - P_3}$

First interation :- Since $(z_1 - c_1)$ and $(z_2 - c_2)$ in the P_1 row are positive, the solution is not optimum and we choose the largest of these. But at the P_1 row both $(z_1 - c_1)$ and $(z_2 - c_2)$ are same, so we chek the next priority levels in x_1 and x_2 columns. We observe that at priority P_2 the largest net evaluation is 2 under x_1 column. Thus the column under x_1 becomes the key column.

Ratio : $\text{Min}\left\{\frac{x_{B1}}{x_{11}}, \frac{x_{B2}}{x_{21}}\right\} \Rightarrow \text{Min}\left\{\frac{400}{1}, \frac{240}{1}\right\} = 240$

* x_1 enter
* d_2^- leave the basis
* x_{21} becomes leading element
* Unity of leading element

Second Iteration :

	$C_j \rightarrow$		0	0	P_1	$2P_2$	P_2	P_3	
C_B	Basic variable	X_B	x_1	x_2 (Enter)	d_1^-	d_2^-	d_3^-	d_1^+	Ratio
P_1	Leave $\rightarrow d_1^-$	160	0	1	1	−1	0	−1	$\frac{160}{1} = 160$
0	x_1	240	1	0	0	1	0	0	
P_2	d_3^-	300	0	1	0	0	1	0	$\frac{300}{1} = 300$
	P_3	0	0	0	0	0	0	−1	
	P_2	300	0	1	0	−2	0	0	
	P_1	160	0	1 (Most positive)	0	−1	0	−1	

Since $(z_2 - c_2) > 0$ at P_1 level, column Under x_2 becomes the key column.

Ratio : $\text{Min}\left\{\frac{x_{B1}}{x_{12}}, \frac{x_{B3}}{x_{32}}\right\} = \text{Min}\{160, 300\} = 160$

* x_2 enter the basis.
* d_1^- leave the basis
* x_{12} becomes a ~~bo~~ leading element.
* Unity of leading element

Third Iteration:

Enter

		$C_j \rightarrow$	0	0	P_1	$2P_2$	P_2	P_3
C_B	Basic variable	X_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+
0	x_2	160	0	1	1	−1	0	−1
0	x_1	240	1	0	0	1	0	0
P_2	d_3^-	140	0	0	−1	1	1	1
$(Z_j - C_j)$	P_3	0	0	0	0	0	0	−1
	P_2	140	0	0	−1	−1	0	1
	P_1	0	0	0	−1	0	0	0

leave

leading element

MOST positive

Since $Z_6 - C_6 > 0$ at P_2 leavel
Column under d_1^+ becomes the key column.

Ratio: $\text{Min}\left\{\frac{X_{B3}}{d_{36}}\right\} = \text{Min}\{140\} = 140.$

- ☆ d_1^+ enter the basis
- ☆ d_3^- leave the basis
- ☆ 3 row, 6th calum i.e $(d_1^+)_{36}$ becomes a leading element.
- ☆ Unity of the leading element

Final Iteration :

		$C_j \rightarrow$	0	0	P_1	$2P_2$	P_2	P_3
C_B	Basic Variable	X_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+
0	x_2	300	0	1	0	0	1	0
0	x_1	240	1	0	0	1	0	0
P_3	d_1^+	140	0	0	-1	1	1	1
$(Z_j - C_j)$	P_3	140	0	0	-1	1	1	0
	P_2	0	0	0	0	-2	-1	0
	P_1	0	0	0	-1	0	0	0

$$Z = C_B \times X_B$$
$$= 0 \times 300 + 0 \times 240 + P_3 \times 140$$
$$\boxed{Z = 140P_3}$$

$$Z_1 - C_1 = 0 \times 0 + 0 \times 1 + P_3 \times 0 - 0$$
$$\boxed{Z_1 - C_1 = 0}$$

$$Z_2 - C_2 = 0 \times 1 + 0 \times 0 + P_3 \times 0 - 0$$
$$\boxed{Z_2 - C_2 = 0}$$

$$Z_3 - C_3 = 0 \times 0 + 0 \times 0 + P_3 \times -1 - P_1$$
$$\boxed{Z_3 - C_3 = -P_3 - P_1}$$

$$Z_4 - C_4 = 0 \times 0 + 0 \times 1 + P_3 \times 1 - 2P_2$$
$$\boxed{Z_4 - C_4 = P_3 - 2P_2}$$

$$Z_5 - C_5 = 0 \times 1 + 0 \times 0 + P_3 \times 1 - P_2$$
$$\boxed{Z_5 - C_5 = P_3 - P_2}$$

,

$$Z_6 - C_6 = 0 \times 0 + 0 \times 0 + P_3 \times 1 - P_3$$
$$Z_6 - C_6 = P_3 - P_3$$
$$\boxed{Z_6 - C_6 = 0}$$

From the above table. We observe that there is positive number at P_3 leave in d_2^-, d_3^- column. But since there is a negative $-2, -1$ at a higher priority leave P_2 in Same column at this cannot become the key column.

therefore third goal cannot be achieved at the expense of achieving the second goal.

Hence the optimum solution is :

$\boxed{x_1 = 240}$

$\boxed{x_2 = 300}$

$\boxed{d_1^+ = 140}$

$\boxed{d_1^- = d_2^- = d_3^- = 0}$

THANK-YOU